Math. ইমদাদ 29-02-2023

8 bit → 1 byte
1024 byte → KB
1024 byte → 1 MB
1024 MB → 1 GB
1024 GB → 1 TB

$f(x)$ $(x-2)$

$R = f(2)$

* Find the remainder when $4x^3 - 5x + 1$ is divided by $x+3$

Let, $f(x) = 4x^3 - 5x + 1$

From Remainder theorem, we have.

$$R = f(-3)$$

$$R = 4(-3)^3 - 5(-3) + 1$$

$$= -27 \times 4 + 15 + 1$$

$$= -108 + 16$$

$$= -92 \text{ A}$$

$$R = 4\left(\frac{1}{2}\right)^3 - 5 \times \frac{1}{2} + 1$$

$$= \frac{4}{8} - \frac{5}{2} + 1$$

$$= 0.5 - 2.5 + 1$$

$$= -2 + 1$$

$$= -1 \text{ B}$$

Give that the exprenion $2x^3 + ax^2 + bx + c$ have the name remainder when divide $x-2$ or $x+1$

Prove that $a + b = -6$

Let, $f(x) = 2x^3 + ax^2 + bx + c$

There are two reminders $f(2)$ a- $f(-1)$ are equ-

$R = f$

6

Math

26-02-16

$f(2) = f(-1)$

$\Rightarrow 2(2)^3 + a\cdot(2)^2 + b(2) + c = 2(-1)^3 + a + b + c$

$\Rightarrow 16 + 4a + 2b + c = -2 + a + b + c$

$\Rightarrow a + b = -6$ (Proved)

(*) When $ax^2 + bx - 6$ is divided by $x+3$; the remainder is 9 Find interms of a only the remainder when $2x^3 - bx^2 + 2ax - 4$ is divided by $x-2$

Let, $f(x) = ax^2 + bx - 6$

$R = f(a)$

$\Rightarrow 9 = f(-3)$

$\Rightarrow a(-3)^2 + b(-3) - 6 = 9$

$\Rightarrow 9a - 3b - 6 = 9$

$\Rightarrow 9a - 3b = 15$

$\Rightarrow 3a - b = 5$

$\Rightarrow b = 3a - 5$

let $g(x) = 2x^3 - bx^2 + 2ax - 4$

$R = g(2)$

$= 2\cdot 2^3 - b\cdot 2^2 + 2a\cdot 2 - 4$

$= 16 - 4b + 4a - 4$

$= 16 - 4(3a-5) + 4a - 4$

Math

29-02-2017

$\Rightarrow 16 - 12a - 20 + 4a - 4$

$\Rightarrow -8a + 32$ Ans

The expression $x^3 + ax^2 + bx - 3$ leave a remainder of 27 when divided by $x-2$ and a remainder of 3 when divided by $x+1$ calculated the remainder when the polynomial is divided by $x-1$.

let $f(x) = x^3 + ax^2 + bx - 3$.

From Remainder Theorem

$f(2) = 27 \Rightarrow 8 + 4a + 2b - 3 = 27$ — (1)

$f(-1) = 3 \Rightarrow -1 + a - b - 3 = 3$ — (2)

$4a + 2b = 27 + 3 - 8$

$= 22$ — (3)

$a - b = 3 + 3 + 1$

$= 7$ — (4)

(3) $\times 1$ + (4) $\times 2$

$4a + 2b = 22$

$2a - 2b = 14$

$6a = 36$

$a = 6$, $b = -1$ Ans

$6 - b = 7$

$-b = 7 - 6$

$b = -1$

Math 6

$f(x) = x^3 + ax^2 + bx - 3$ 29-02-2020

$= x^3 + 6x^2 - 8x - 3$

$f(-1) = (-1)^3 + 6 \cdot 1 + 1 - 3$

~~$= -1 + 6 + 1 - 3$~~

~~$= 3 \cdot 3$~~ Assi

$= 1$

A(6). Find the value of k if $(3x+k)^3 + (4x-7)^2$ has a remainder of 33 when divided by $(x-3)$.

A-(7) The expression $x^3 + ax^2 + 7$ leaves a remainder of $2p$ when divided by $x+1$ and a remainder of $p+5$ when divided by $(x-2)$. Calculate the value of a and p.

Math. 29-03-2019

Factor Theorem.

iff and and only iff

(*) $x-a$ is a factor of the polynomial $f(x) \Leftrightarrow f(a)=0$

Determine whether or not $(x+1)$ is a factor of the polynomials;

(i) $3x^4+x^3-x^2+3x+2$ (ii) $x^6+2x(x-1)-4$

Let, $f(x) = 3x^4+x^3-x^2+3x+2$

$f(-1) = 0$

$x+1$ is a factor of $f(x)$

Let $g(x) = x^6+2x(x-1)-4$

$g(-1) = 1$

$(x+1)$ is not a factor of $g(x)$

(*) The expenion ax^3-8x^2+bx+6 is exactly divisible by x^2-2x-3. Calculate the value of a and b.

$x^2-2x-3=0$

$\Rightarrow x^2-3x+x-3=0$

$x(x-3)+1(x-3)=0$

$(x-3)(x+1)=0$

$x=3$

$x=-1$

$f(3)=0 \Rightarrow 9a+b=22$

$f(-1)=0 \Rightarrow a+b=-2$

$a=3$

$b=-5$

29-07-26

(*) The polynomial $2x^3+x^2+px-4$ has a factor $x-2$, find p. Show that $(2x+1)$ is also a factore and deduce the third factore.

Math:

$f(x) = 2x^3+x^2+px-4$

$f(2) = 2\times2^3+2^2+p.2-4 = 0$ [∵ from Factor theorem]

$\Rightarrow 16+4+2p-4=0$

$\Rightarrow 2p+16=0$

$\Rightarrow p=-8$

$f(-\frac{1}{2}) = 2x^3+x^2-8x-4$

$= 0$

$2x^3+x^2-8x-4$

$= 2x^3-4x^2+5x^2-10x+2x-4$

$= 2x^2(x-2)+5x(x-2)+2(x-2)$

$= (x-2)(2x^2+5x+2)$

$= (x-2)(2x^2+4x+x+2)$

$= (x-2)\{2x(x+2)+1(x+2)\}$

$= (x-2)(x+2)(2x+1)$

Math; 29-03-26

Ass → (3)

Let; $f(x) = (3x + k)^3 + (4x - 7)^2$

From Remainder Theorem,

$f(x) = 33$

$f(3) = 33$

$\Rightarrow (3.3 + k)^3 + (4.3 - 7)^2 = 33$

$\Rightarrow (9 + k)^3 + (12 - 7)^2 = 33$

$\Rightarrow (9 + k)^3 + 25 - 33 = 0$

$\Rightarrow (9 + k)^3 = 8$

$\Rightarrow 9 + k = \sqrt[3]{8}$

$\Rightarrow k = -7$ (Ans)

Ass → (4)

Let, $f(x) = x^3 + ax^2 + 7$

From Remainder Theorem,

$f(-1) = 2P$

$\Rightarrow (-1)^3 + a(-1)^2 + 7 = 2P$

$\Rightarrow -1 + a + 7 = 2P$

$\Rightarrow a - 2P = -6$ —— (i)

Again, $f(2) = P + 5$

$\Rightarrow 2^3 + a.2^2 + 7 = P + 5$

$\Rightarrow P - 4a = 10$ —— (ii)

(i) + (ii) × 2 →

$a - 2P = -6$

$-8a + 2P = 20$

$-7a = 14$

$\therefore a = -2$

(ii) No. a →

$P = 10 - 8$

$= 2$

$\therefore a = -2, P = 2$ } Ans

(A-5)

Let $f(x) = x^2 - 2x - 3$

$= x(x-3) + 1(x-3)$

$= (x-3)(x+1)$

$\therefore f(3) = 0$

$\Rightarrow 27a - 8 \times 6 + 3b + 6 = 0$

$\Rightarrow 27a + 3b = 42$

$\Rightarrow 9a + b = 22$ —— (i)

$f(-1) = 0$

$\Rightarrow a(-1)^3 - 8(-1)^2 + b(-1) + 6 = 0$

$\Rightarrow -a - 8 + 6 - b = 0$

$\Rightarrow a + b = -2$ —— (ii)

(i) − (ii) →

$9a + b = 22$

$-a \pm b = \mp 2$

$8a = 24$

$\therefore a = 3$

$b = -5$

Computer

Lecture-2/3 - Sec 2A/2B.

Alternative Methods of Input.

M.T Abdur ABDUR RAHMAN

R;

Math

The expression $ax^3 + bx^2 - 5x + 2a$ is exactly divisible by $x^2 - 3x - 4$. calculate the value of a and of b and factorise the expression completely

$x^2 - 3x - 4$

$= x^2 - 4x + x - 4$

$= x(x-4) + 1(x-4)$

$= (x-4)(x+1)$

$f(4) \Rightarrow 64a + 16b - 20 + 2a = 0$

$\Rightarrow 66a + 16b = 20$ —— (I)

$f(2) \Rightarrow 8a + 4b - 10 + 2a = 0$

$\Rightarrow 10a + 4b = 10$ —— (II)

(I) − (II) × 4 →

$66a + 16b = 20$

$40a + 16b = 40$

$- \quad - \quad -$

$26a = -20$

$a = 2$
$b = -7$ } Ans

$2x^3 - 7x^2 - 5x + 4$

$= 2x^3 + 2x^2 - 9x^2 - 9x + 4x + 4$

$= 2x^2(x+1) - 9x(x+1) + 4(x+1)$

$= (x+1)(2x^2 - 9x + 4)$

$= (x+1)(2x^2 - 8x - x + 4)$

$= (x+1)\{2x(x-4) - 1(x-4)\}$

$= (x-4)(x+1)(2x-1)$ Ans

A-5. Given that $(x+2)$ and $(3x-1)$ are factors of the expression $3x^3 - 10x^2 + px + q$. find the value of p and of q. With these values of p and q, find the third factor of the expression.

A-6 If $(x-2)$ and $(x+3)$ are factors of the expression $3x^4 + 5x^3 - px^2 + qx + 6$, find the values of p and of q.

Rx (*) Solving Cubic equation.

$ax^2 + bx + c$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad x^2(5-2x) = 4$$

(*) Solve the equation $x^3 + 4x^2 - 23x + 6 = 0$, giving solution to 2 decimal places where necessary.

$$f(x) = x^3 + 4x^2 - 23x + 6$$

$$f(3) = 3^3 + 4.3^2 - 23\times3 + 6$$

$$= 0$$

$(x-3)$ is a factor of $f(x)$

$$x^3 + 4x^2 - 23x + 6$$

$$= x^3 - 3x^2 + 7x^2 - 21x - 2x + 6$$

$$= x^2(x-3) + 7x(x-3) - 2(x-3)$$

$$= (x-3)(x^2 + 7x - 2) = 0$$

$a = 1$
$b = 7$
$c = -2$

$= ~~(x-3)~~$

$$= x = \frac{-7 \pm \sqrt{49 - 4.1.(-2)}}{2.1}$$

$$x = \frac{-7 \pm \sqrt{57}}{2}$$

$$= +0.27, -7.27, 3$$ (Ans)

let $f(x) = x^2(5-2x) - 4$

$f(2) = 0$

$(x-2)$ is a factor of $f(x)$

$a=2$
$b=-1$
$c=-2$

$5x^2 - 2x^3 - 4 = 0$

$\Rightarrow 2x^3 - 5x^2 + 4 = 0$

$\Rightarrow 2x^3 - 4x^2 - x^2 + 2x - 2x + 4$

$\Rightarrow 2x^2(x-2) - x(x-2) - 2(x-2)$

$= (x-2)(2x^2 - x - 2)$

$x = 2,$

$$x = \frac{+1 \pm \sqrt{1 - 4 \times 2 \cdot (-2)}}{2 \cdot 2}$$

$$= \frac{1 \pm \sqrt{1+16}}{4}$$

$$= \frac{1 \pm \sqrt{17}}{4}$$

$= 1.28, -0.78, 2$ Ans

A-7. Solve the equation $4x^3-16x^2+15x-4=0$ giving solution to 2 decimal place where necessary

$$3x^3-23x^2+7x+5=0$$

$$f(-\tfrac{1}{3})=0$$

$$a+\tfrac{1}{3}=0$$

$$\Rightarrow 3x^3-23x^2+7x+5=0$$

$$\Rightarrow 3x^3+x^2-24x^2-8x+$$

$$= x^2(3x+1)-8x(3x+1)$$

$(3x+1)$

A-5

Let $f(x) = 3x^3 - 10x^2 + px + q$

$f(-2) = 3(-2)^3 - 10(-2)^2 + p(-2) + q = 0$

$\Rightarrow -2p + q = 64$ —— (1)

$f(\frac{1}{3}) = 3(\frac{1}{3})^3 - 10(\frac{1}{3})^2 + p(\frac{1}{3}) + q = 0$

$\Rightarrow \frac{p}{3} + q = 1$ —— (2)

(1) − (2) →

$-\frac{7}{3}p = 63$

$\Rightarrow p = -27$

$100 — 15$

$1 — \frac{15 \times 275}{100}$

(2) No p value →

$q = 10$

$\therefore 3x^3 - 10x^2 - 27x + 10$

$= 3x^3 - x^2 - 9x^2 + 3x - 30x + 10$

$= x^2(3x-1) - 3x(3x-1) - 10(3x-1)$

$= (3x-1)(x^2 - 3x - 10)$

$= (3x-1)\{x(x-5) + 2(x-5)\}$

$= (3x-1)(x-5)(x+2)$

A-6

Let $f(x) = 3x^4 + 5x^3 + px^2 + qx + 6$

$f(2) = 0$

$\Rightarrow 4p + 2q = -94$

$\Rightarrow 2p + q = -47$ —— (i)

$f(3) = 0$

$\Rightarrow 3p + q = -128$ —— (ii)

(i) − (ii)

$p = -81$

(ii) No → $q = 115$

$\therefore p = -81$, $q = 115$

(A-7) Let ~~f(x) =~~ $4x^3 - 16x^2 + 15x - 4 = 0$

$\Rightarrow 4x^3 - 2x^2 - 14x^2 + 7x + 8x - 4 = 0$

$\Rightarrow 2x^2(2x-1) - 7x(2x-1) + 4(2x-1) = 0$

$\Rightarrow (2x-1)(2x^2 - 7x + 4) = 0$

$\therefore x = \frac{1}{2}, \quad x = \frac{7 \pm \sqrt{(-7)^2 - 4\times2\times4}}{2.2}$

$= 2.78, 0.72$

$\therefore x = \frac{1}{2}, 0.72, 2.78$ Ans

$3.2^4 + 5.2^3 - P.2^2 + 9.2 + 6 = 0$

$\Rightarrow 29 - 4P = -94$

$9 - 2P = -47$

$2P - 9 = 47$

$39 \quad -9P = -114$

$2P - 9 = 47$

$3P - 9 = 38$

$-P = 9$

$P = -9$

$-18 - 9 = 47$

$-9 = 47 + 18$

=

Rx **Math** 30-01-13

Rational number. $\frac{2}{3}$, $\frac{4}{5}$, $\frac{6}{7}$

Irrational number, $3\sqrt{3}$

An irrational number involving a root is called Surd

Simplify, $5\sqrt{75}$

$$\sqrt{243} - \sqrt{12} + 2\sqrt{75}$$

$$= \sqrt{81 \times 3} - 2\sqrt{3} + 2\sqrt{3 \times 25}$$

$$= 9\sqrt{3} - 2\sqrt{3} + 10\sqrt{3}$$

$$= \sqrt{3}\,(9 - 2 + 10)$$

$$= 17\sqrt{3} \text{ Ans}$$

(A) $\sqrt{112} - \sqrt{63} + \frac{224}{\sqrt{28}}$ $\sqrt{256 \times 7}$

$$= \sqrt{16 \times 7} - \sqrt{9 \times 7} + \sqrt{}$$

$$= 4\sqrt{7} - 3\sqrt{7} + 16\sqrt{7}$$

$$= 17\sqrt{7}$$

$$\frac{4\sqrt{3} - 3\sqrt{2}}{2\sqrt{3} - 3\sqrt{2}}$$

$$= \frac{(4\sqrt{3} - 3\sqrt{2})(2\sqrt{3} + 3\sqrt{2})}{(2\sqrt{3} - 3\sqrt{2})(2\sqrt{3} + 3\sqrt{2})}$$

$$= \frac{8\times 3 + 12\sqrt{6} - 6\sqrt{6} - 9\times 2}{4\times 3 - 9\times 2}$$

$$= \frac{24 + 12\sqrt{6} - 6\sqrt{6} - 18}{12 - 18}$$

$$= \frac{6 + 6\sqrt{6}}{-6}$$

$$= -1 - \sqrt{6}.$$

$7\sqrt{7}$ $\quad \frac{7\sqrt{3} + 2\sqrt{5}}{3\sqrt{5} - 2\sqrt{3}}$

$$= \frac{(7\sqrt{3} + 2\sqrt{5})(3\sqrt{5} + 2\sqrt{3})}{(3\sqrt{5})^2 - (2\sqrt{3})^2}$$

$$= \frac{21\sqrt{15} + 14\times 3 + 6\times 5 + 4\sqrt{15}}{9\times 5 - 4\times 3}$$

$$= \frac{72 + 25\sqrt{15}}{33}$$

Indices.

$a \rightarrow$ base

$n \rightarrow$ power

$a^{\frac{1}{n}} = \sqrt[n]{a}$

$\frac{1}{5^3}$

$8^{\frac{2}{3}} = \sqrt[3]{8^2}$

$\sqrt[4]{16^3}$

$= 8$

$16^{3/4}$

$\sqrt[4]{16^3}$

$= \sqrt[4]{8} = 2^3$

$\sqrt[2]{25^{-3}}$

$25^{-3/2}$

$\frac{1}{25^{3/2}}$

$= \frac{1}{5^3}$

$= \frac{1}{25}$

$\sqrt{x+4}$

$(0.6)^{x-3} = (3.2)$

$(x-3) \log 0.6 = (x+4) \log 3.2$

$\frac{\sqrt[3]{a} \times \left(\frac{1}{a}\right)^{\frac{1}{2}}}{5\sqrt{a^2}}$

$x = -1.86$

$4 (5^{2x})$

$2^{2x} \cdot 5^{2x} = (10)^{2x} = 10$

$2x = 1$

$x = \frac{1}{2}$

$\sqrt[3]{a} \div \sqrt[5]{a^2} \times \left(a^{-1}\right)^{\frac{1}{2}}$

$= a^{\frac{1}{3}} \div a^{\frac{2}{5}} \times a^{-\frac{1}{2}}$

$= a^{\frac{1}{3} - \frac{5}{2} - \frac{1}{2}}$

$= a^{-8/3}$

$= a^{-\frac{17}{30}}$

$=$

1 - 4 3

5 - 9 4

4 - 5 □

$$\left(a^{-\frac{1}{4}} \div a^{\frac{3}{8}}\right)^{24}$$

$$= \left(a^{-\frac{1}{4}-\frac{3}{8}}\right)^{24}$$

$$= \left(a^{\frac{-2-3}{8}}\right)^{24}$$

$$= \left(a^{\frac{-5}{8}}\right)^{24}$$

$$= a^{-\frac{5\times 24}{8}}$$

$$= a^{-15}$$

$$\sqrt[6]{a^{x+2}} \div \sqrt[9]{a^{x+3}}$$

$$= a^{\frac{x+2}{6}} \div a^{\frac{x+3}{9}}$$

$$= a^{\frac{x+2}{6} - \frac{x+3}{9}}$$

$$= a^{\frac{3x+6-2x-6}{18}}$$

$$= a^{\frac{x}{18}}$$

$$3\underline{|\,6, 9}$$
$$\;\;2, 3$$

Exponential equation

$$a^x = a^n$$
$$x = n$$

$$2^x = 32$$

$$2^x = 2^5$$

$$x = 5$$

$$9^{2x+1} = 27^x$$

$$3^{6x+2} = 3^{3x}$$

$$6x + 2 = 3x$$

$$3x = -3$$

$$x = -2$$

$$\frac{4^x}{2^{x-1}} = 8^{2-x}$$

$$2^{2x-x+1} = 2^{6-3x}$$

$$x+1 = 6-3x$$

$$4x = 6-1$$

$$x = \frac{5}{4}$$

$a^n \times a^m$ a^{n+m} $a^n + a^m$

$$5^{x^2-1} = 5^0$$

$$x^2-1 = 0$$

$$x = \pm 1$$

$$2^x \cdot 2^x \times 2^3 + 2^x \times 2^3 = 1 + 2^x \qquad y = 2^x$$

$$\Rightarrow 8y^2 + 8y = 1 + y$$

$$\Rightarrow (8y-1)(y+1) = 0$$

$$y = \frac{1}{8}, -1$$

when $y = \frac{1}{8}$

$$2^x = \frac{1}{8} = 2^{-3} \quad x = -3$$

$y = -1$

$2^x = -1$

$2^x = -2$

$x = 0$ it is not acceptable

$3^{2x+2} + 81 = 246(3^x)$ $\qquad 3^x = y$

$\Rightarrow 3^x \cdot 3^x \cdot 3^2 + 3^4 = 246y$

$\Rightarrow 9y^2 - 246y + 81 = 0$

~~$\Rightarrow 3y^2 - 82y + 27 = 0$~~

$a = 3$, $b = -82$, $c = 27$

$3 \mid 81$, $3 \mid 27$, $3 \mid 9$, 3

$-245y - y$

$\Rightarrow 3y^2 - 27 \times 3y - y + 27 = 0$

$\Rightarrow (3y - 1)(y - 27) = 0$

$\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$y = \frac{1}{3}, 27$

$= \dfrac{82 \pm \sqrt{82^2 - 4}}{}$

$y = 3^x$

$\frac{1}{3} = 3^x$

$3^{-1} = 3^x$

$x = -1$

$3^x = 3^3$

$x = 3$

(A8) Evaluate each of the following without using a calculator :

ⓐ $\left(\frac{1}{625}\right)^{-1/4} \times 4^{-3}$

ⓑ $\sqrt[3]{\dfrac{8a^{-3}}{64b^{-3}}}$

ⓒ $5^{3x} \div 25^{x+1} = \frac{1}{125}$

ⓓ $3^{2t+1} + 3^{t+2} = 3^{1/3}$

A-8 (a) $\left(\frac{1}{625}\right)^{-\frac{1}{4}} \times 4^{-3}$

$= \frac{1}{\left(\frac{1}{625}\right)^{\frac{1}{4}}} \times \frac{1}{64}$

$= (625)^{\frac{1}{4}} \times \frac{1}{64}$

$= \sqrt[4]{625} \times \frac{1}{64}$

$= \frac{5}{64}$ (Ans)

(b) $\sqrt[3]{\frac{8a^{-3}}{64b^{-3}}}$

$= \frac{2(a^{-3})^{\frac{1}{3}}}{4(b^{-3})^{\frac{1}{3}}}$

$= \frac{a^{-1}}{2b^{-1}}$

$= \frac{b}{2a}$ (Ans)

(c) $5^{3x} \div 25^{x+1} = \frac{1}{125}$

$\Rightarrow 5^{3x} \div 5^{2x+2} = \frac{1}{5^3}$

$\Rightarrow 5^{3x-2x-2} \times 5^3 = 1$

$\Rightarrow 5^{x-2+3} = 5^0$

$\Rightarrow 5^{x+1} = 5^0$

$x = -1$ Ans

Standard Output 1½. 3PM. 04.2.13

No

(a) $3^{2t+1} + 3^{t+2} = 3\frac{1}{3}$ $\quad 3 \cdot \frac{1}{3} \quad \frac{10}{3}$

$\Rightarrow 3y^2 + 9y = 3\frac{1}{3}$

$\Rightarrow 9y^2 + 27y - 10 = 0$

$\Rightarrow 9y^2 + 30y - 3y - 10 = 0$

$\Rightarrow (3y-1)(3y+10)$

$\therefore y = \frac{1}{3}$

$3^t = 3^{-1}$

$t = -1$ $\quad 4^2 = 2^4$

(b) $2^{2x} 2^y = 16 \Rightarrow 2x + y = 4$ — (i)

$3^{xy - 3x} = 3^{-1}$

$y - 3x = -1$ — (ii)

$2x + y = 4$

$-3x + y = -1$

$\underline{+ \quad - \quad +}$

$5x = 5$

$x = 1$

$2 \cdot 1 + y = 4$

$y = 2$

(A-9) Given that $y = ax^b + 10$, and that $y = 26$ when $x = 2$ and $y = 64$ when $x = 3$, find the value of a and of b, (a = 2, b = 3)

$y = a^x$ means that $\log_a y = x$ [[illegible] a]

Find the value of $\log_2 64$

$\log_2 64 = x$

$\Rightarrow 64 = 2^x$

$\Rightarrow 2^6 = 2^x$

$\log_8 0.25 = x$

$\Rightarrow 8^x = 0.25$

$\Rightarrow 2^{3x} = \frac{1}{2^2}$

$\Rightarrow 2^{3x} = 2^{-2}$

$3x = -2$

$x = -\frac{2}{3}$

Evaluate 1.26

(A-1) Solve the equation giving your answer correct answer correct to 3 significant figure. $(0.6)^{x-3} = 3.2^{x+4}$

[9]

$\log_{10}\sqrt{175} - \log_{10}\sqrt{91} + \log_{10}\sqrt{52}$

$= \log_{10} 5\sqrt{7} - \log_{10} \sqrt{7}\cdot\sqrt{13} + \log_{10} 2\sqrt{13}$

$= \log_{10} \frac{5\sqrt{7}\times 2\sqrt{13}}{\sqrt{7}\times\sqrt{13}}$

$= \log_{10} 10$

$= 1$

$\frac{2}{3}\log_2 8 - \frac{3}{2}\log_2 16 + \frac{1}{2}\log_2 32$

$\frac{2}{3}\log_2 2^3 - \frac{3}{2}\log_2 2^4 + \frac{1}{2}\log_2 2^5$

$\Rightarrow \frac{2}{3}\times 3 - \frac{3}{2}\times 4 + \frac{1}{2} 5 \quad = 2 - 6 + \frac{5}{2}$

$= \frac{4-12+5}{2}$

$= -3/2$

4×8

$2^2 \cdot 3^3$

A-10

$\frac{1}{2}\log_{10}\frac{81}{17} - \log_{10}\frac{17}{4} + 2\log_{10}\frac{5}{3} + \frac{3}{2}\log_{10} 17$ Ans - 2

Solved the equation:-

$\log_{10}(3x+2) - 2\log_{10} x = 1 - \log_{10}(5x-3)$

$\Rightarrow \log_{10}\frac{(3x+2)(5x-3)}{x^2} = 1$

$\Rightarrow 10^1 = \frac{(3x+2)(5x-3)}{x^2}$

$\Rightarrow 5x^2 + x - 6 = 0$

$\Rightarrow 5x^2 + 3x - 2x - 6$

$\Rightarrow 5(5x+6) \quad (5x+6)(5x-1)$

$5x = -6/5 \quad x = 1$

$\log_5 x = 4\log_x 5$

$\log_a b = \frac{1}{\log_b a}$

$\Rightarrow \frac{1}{\log_x 5} = 4\log_x 5 \Rightarrow \log_5 x = \frac{4 \cdot 1}{\log_5 x}$

$\Rightarrow (\log_5 x)^2 = 4$

$\Rightarrow \log_5 x = \pm 2$

$\Rightarrow 5^2 = x$

$x = 25$

$5^{-2} = x$

$\frac{1}{25} = x$

বুধবার - ১১ টায় (অংক আগের class,

$6^{x+2} = 21$

$\log_{10} 6^{x+2} = \log_{10} 21$

$\log_6 x+2 = \log 21 = \log 21$

~~$x+2 \log_{10} 6 \quad \log_{10} 3 = 0$~~

$x + 2 = \frac{\log 21}{\log 6}$

$x = -0.301$

Solve the eq.

$2(\log_2 x)^2 = 7\log_2 x - 3$

$\Rightarrow$ ~~$4\log_2 x - 7\log_2 x = 3$~~

$\Rightarrow 2P^2 - 7P + 3 = 0$

$\Rightarrow 2P^2 - 6P - P + 3 = 0$

$\Rightarrow 2P(P - 3) - 1(P - 3) = 0$

$P = 3$

$P = \frac{1}{2}$

$\log_2 x = 3$

$2^3 = x$

$x = 8$

$\log_2 x = \frac{1}{2}$

$2^{\frac{1}{2}} = x$

$x = \sqrt{2}$

Other high Quality Printers 12-3'00 4'30

- plotter (Inkjet printer) 1 — 5

11.2.13 18.02.13

Express the folling equation in the form $\ln x = ax+b$ and find a and find b.

$$x^3 = e^{6x-1}$$

$$\Rightarrow 3\ln x = (6x-1)\ln e.$$

$$\Rightarrow \ln x = \frac{6x-1}{3}$$

$$= 2x - \frac{1}{3}$$

$a = 2$

$b = -\frac{1}{3}$

$$xe^{-x} = 2.46$$

$$~~\ln xe^{-x} = \ln(2.46)~~$$

$$\Rightarrow \ln x + \ln e^{-x} = \ln 2.46$$

$$\Rightarrow \ln x - x = \ln 2.46$$

$$\Rightarrow \ln x = 0.900 + x$$

$$\Rightarrow$$

$a = 1$

$b = 0.9$

A-12 $(xe^{x})^{2} = 30e^{-x}$

~~ln (xe^x~~ **Matrix**

We can represent the number as a rectangular array of numbers.

The following tables shows the stock of pens at two stationary shops.

Red leaf
70
54

	Pilot	Penter	Red leaf
Shop 1	94	~~94~~ 104	~~70~~
Shop 2	65	~~65~~ 83	54

Order

Order 2 x 3 (rows, cols)

m x n

Equation

Equality of matrices:-

Two matrices are equal if they have Same order and if their corresponding elements are equal

$$\begin{pmatrix} a & b \\ c & 21 \end{pmatrix} = \begin{pmatrix} 1 & -5 \\ 4 & 21 \end{pmatrix}$$

$$IA = AI = A$$

Square matrices,

no of row = no of colum.

Identity matrix:-

Addition, Subtraction of the matrices.

$$A+B = \begin{pmatrix} 4+3 & 2+9 & 6+0 \\ 1+4 & 3-1 & -7+8 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & 11 & 6 \\ 5 & 2 & 1 \end{pmatrix}$$

$$B - C = \begin{pmatrix} 1 & 4 \\ & -5 \end{pmatrix} \begin{matrix} 0 \\ 0 \end{matrix}$$

does not allowed.

If $D = \begin{pmatrix} p & -7 & q \\ r & 4 & -15 \end{pmatrix}$ and $D + B = A$

~~D+B =~~ ~~R = 3~~ $q = 0$

~~-7~~ ~~r = 4~~

~~D+B =~~ $D + B = \begin{pmatrix} p+3 & -9+7 & q+0 \\ r-4 & & \end{pmatrix}$

~~R+3 = 4~~

~~p = 4 - 3~~

~~= 1~~

$G = \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix}$ $F = \begin{pmatrix} 4 & 5 \end{pmatrix}$

$GF = \begin{pmatrix} 16 & 20 \\ 4 & 5 \\ 12 & 15 \end{pmatrix}$ (3×2) Ans

Find the value of each

$\begin{pmatrix} 1 & 3 \end{pmatrix} \begin{pmatrix} p & 2 \\ -1 & q \end{pmatrix} = \begin{pmatrix} 7 & 11 \end{pmatrix}$

1×2 2×2

~~$\begin{pmatrix} p & 6 \\ -1 & 3p \end{pmatrix}$~~

$\begin{pmatrix} p - 3 & 2 + 3q \end{pmatrix} = \begin{pmatrix} 7 & 11 \end{pmatrix}$

$p = 10$

$3q = 11 - 2$

$= 3$

$p = 10$

$q = 3$ Ans

Scalar Multiplication

KA

Multiplication:

$$\underset{m\times e}{A} \quad \underset{e\times t}{B} = \underset{m\times t}{P}$$

equal

$$C = \begin{pmatrix} 4 & 3 \\ 1 & -2 \end{pmatrix}, \quad K = \begin{pmatrix} 1 & -3 \\ 0 & 1 \\ -7 & 0 \end{pmatrix}$$

CK =

2×2 3×2

$$H = \begin{pmatrix} 0 & 7 & -2 \\ 3 & -4 & 5 \end{pmatrix}$$

$$K = \begin{pmatrix} 1 & -3 \\ 0 & 1 \\ -7 & 0 \end{pmatrix}$$

HK = (~~0 + 0 + 14~~ ~~-9 + 4 + 0~~) 1

= (4 ~~-13~~)

$$HK = \begin{pmatrix} 14 & 1 \\ 3 - 35 & -9 - 4 \end{pmatrix}$$

$$= \begin{pmatrix} 14 & 1 \\ -32 & -13 \end{pmatrix}$$

Two matrix (A, B) have same order

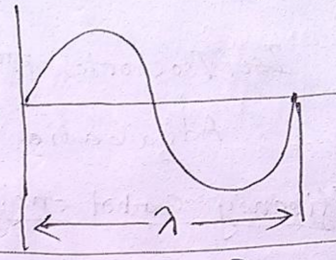
Waves and oscillations

ms^{-2} $g = 9.8$ MKS

cms^{-2} $g = 981$ CGS

FS^{-2} $g = 32$ FKS

Abdur Rahman



$$M = \begin{pmatrix} 1 & S \\ \pi & 6 \end{pmatrix} \qquad N = \begin{pmatrix} 2 & -3 \\ 0 & 8 \end{pmatrix}$$

$$4M = \begin{pmatrix} 4 & 4S \\ 4\pi & 24 \end{pmatrix}$$

$$3N = \begin{pmatrix} 6 & -9 \\ 0 & 24 \end{pmatrix}$$

$$4M - 3N = \begin{pmatrix} 4 & 4S \\ 4\pi & 24 \end{pmatrix} - \begin{pmatrix} 6 & -9 \\ 0 & 24 \end{pmatrix}$$

~~$4S = -9$~~

$$= \begin{pmatrix} 4-6 & 4S+9 \\ 4\pi - 0 & 24-24 \end{pmatrix}$$

$$= \begin{pmatrix} -2 & 4S+9 \\ 4\pi & 0 \end{pmatrix}$$

(c) $NM = \begin{pmatrix} 2 & -3 \\ 0 & 8 \end{pmatrix} \begin{pmatrix} 1 & s \\ r & 6 \end{pmatrix}$

$= \begin{pmatrix} 2-3r & 2s-18 \\ 8r & 48 \end{pmatrix}$

$= 8M$

$8M = \begin{pmatrix} 8 & 8s \\ 8r & 48 \end{pmatrix}$

$2-3r = 8$

$r = -2$

$2s-18 = 8s$

$s = -3$

If $A = \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$, $B = \begin{pmatrix} x & 0 \\ 1 & 3 \end{pmatrix}$

$$AB = \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x & 0 \\ 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} x+0 & 0+0 \\ 3x+2 & 0+6 \end{pmatrix}$$

$$= \begin{pmatrix} x & 0 \\ 3x+2 & 6 \end{pmatrix}$$

$$BA = \begin{pmatrix} x & 0 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} x+0 & 0+0 \\ 1+9 & 0+6 \end{pmatrix}$$

$$= \begin{pmatrix} x & 0 \\ 10 & 6 \end{pmatrix}$$

$x = x$

$3x+2 = 10$

$x = 8$

$3x+2$

$AB = \begin{pmatrix} x+0 & 0+0 \\ 3x+2 & 0+6 \end{pmatrix} = \begin{pmatrix} x & 0 \\ 3x+2 & 6 \end{pmatrix}$

$B.A = \begin{pmatrix} x & 0+0 \\ 1+9 & 0+6 \end{pmatrix} = \begin{pmatrix} x & 0 \\ 10 & 6 \end{pmatrix}$

$3x+2 = 10$

$3x = 8$

$x = 8/3$

$$A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$A = \begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 5 & 0 \\ -2 & 4 \end{pmatrix}$$

$$A^{-1} = \frac{1}{3\times 0 + 1\times 2}\begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix}$$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$AX = B$

$X = \frac{B}{A}$

$(A^{-1}A)X = BA^{-1}$

$I = BA^{-1}$

$AX = B$

$AX = A^{-1}B$

$$= \frac{1}{2}\begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 5 & 0 \\ -2 & 4 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 0-2 & 0+4 \\ -10+6 & 0+12 \end{pmatrix}$$

$$= \begin{bmatrix} -1 & 2 \\ -8 & 6 \end{bmatrix}$$

Use a matrix method to solved the simultaneous equation.

$$3x+2y=8$$
$$x-y=6$$

Simultaneous equation can be written as the matrix equation

$$\begin{pmatrix}3 & 2\\ 1 & -1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}8\\ 6\end{pmatrix}$$

$$A^{-1}=\frac{1}{ad-bc}\begin{pmatrix}d & -b\\ -c & a\end{pmatrix}$$

$$\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}8\\ 9\end{pmatrix}\frac{1}{3(-1)-2\times1}\begin{pmatrix}-1 & -2\\ 1 & 3\end{pmatrix}$$

$$=\begin{pmatrix}8\\ 9\end{pmatrix}\frac{1}{-3-2}\begin{pmatrix}-1 & 2\\ 1 & 3\end{pmatrix}$$

$$=\begin{pmatrix}8\\ 9\end{pmatrix}\left(\frac{1}{-5}\right)\begin{pmatrix}-1 & 2\\ 1 & 3\end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$$

$$B = \begin{pmatrix} x & 0 \\ 1 & 3 \end{pmatrix}$$

$$A \cdot B = \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x & 0 \\ 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} x + 0 & 0 + 0 \\ 3x + 2 & 0 + 6 \end{pmatrix}$$

$$= \begin{pmatrix} x & 0 \\ 3x+2 & 6 \end{pmatrix}$$

$$B \cdot A = \begin{pmatrix} x & 0 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} x + 0 & 0 + 0 \\ 1 + 9 & 0 + 6 \end{pmatrix}$$

x

Operating system
Basics

Lect-10.

(Accep-L-9)

$$A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

(**)

$$\begin{pmatrix} 3 & 2 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$$

$$A^{-1} = \frac{1}{-5}\begin{pmatrix} -1 & -2 \\ -1 & 3 \end{pmatrix}$$

$$-\frac{1}{5}\begin{pmatrix} -1 & -2 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = -\frac{1}{5}\begin{pmatrix} -1 & -2 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 8 \\ 6 \end{pmatrix}$$

~~$\Rightarrow -\frac{1}{5} I \begin{pmatrix} 3 & 2 \\ 1 & -1 \end{pmatrix}$~~

~~R.S = $-\frac{1}{5}$ (~~

$$L.S = -\frac{1}{5}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix}$$

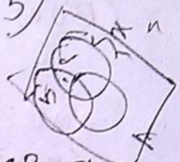
$$R.S = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

$x = 4$
$y = -2$ (Ans)

(140) $N = \begin{pmatrix} 205 & 160 & 70 \\ 310 & 200 & 65 \end{pmatrix}$ $c = \begin{pmatrix} 3 \\ 8 \\ 5 \end{pmatrix}$

615
930
1650
325

$NC = \begin{pmatrix} ~~1895~~ 2245 \\ 2855 \end{pmatrix}$



$\{x \in R, x$
$\{(x,y) : x \in (A \cap B), y \in$

Set theory

Given that $\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{x : x \text{ is a multiple of } 2\}$

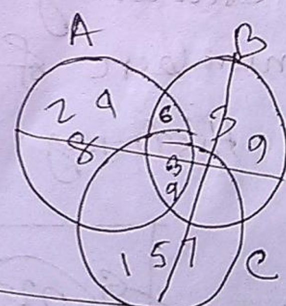
$B = \{x : x \text{ is a multiple of } 3\}$

$C = \{x : x \text{ is odd}\}$

By drawin a venn diagram list the elements of the following lets:

(a) $A \cup B \cup C$

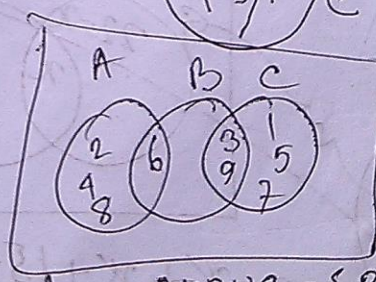
(b) $(B \cup C)'$



$A = \{2, 4, 6, 8\}$

$B = \{3, 6, 9\}$

$C = \{1, 3, 5, 7, 9\}$



$A \cup B \cup C = \{2, 4, 6, 8\} \cup \{3, 6, 9\} \cup \{1, 3, 5, 7, 9\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} = \mathcal{E}$

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$B \cup C = \{1, 3, 5, 6, 7, 9\}$

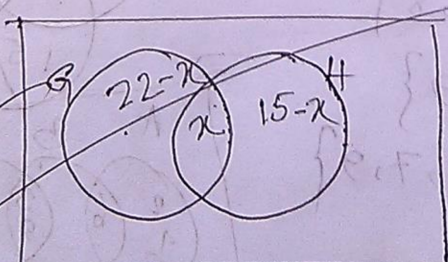
$(B \cup C)' = U - \{1, 3, 5, 6, 7, 9\}$

$= \{2, 4, 8\}$

$n(B \cup C)' = 3$

Set theory

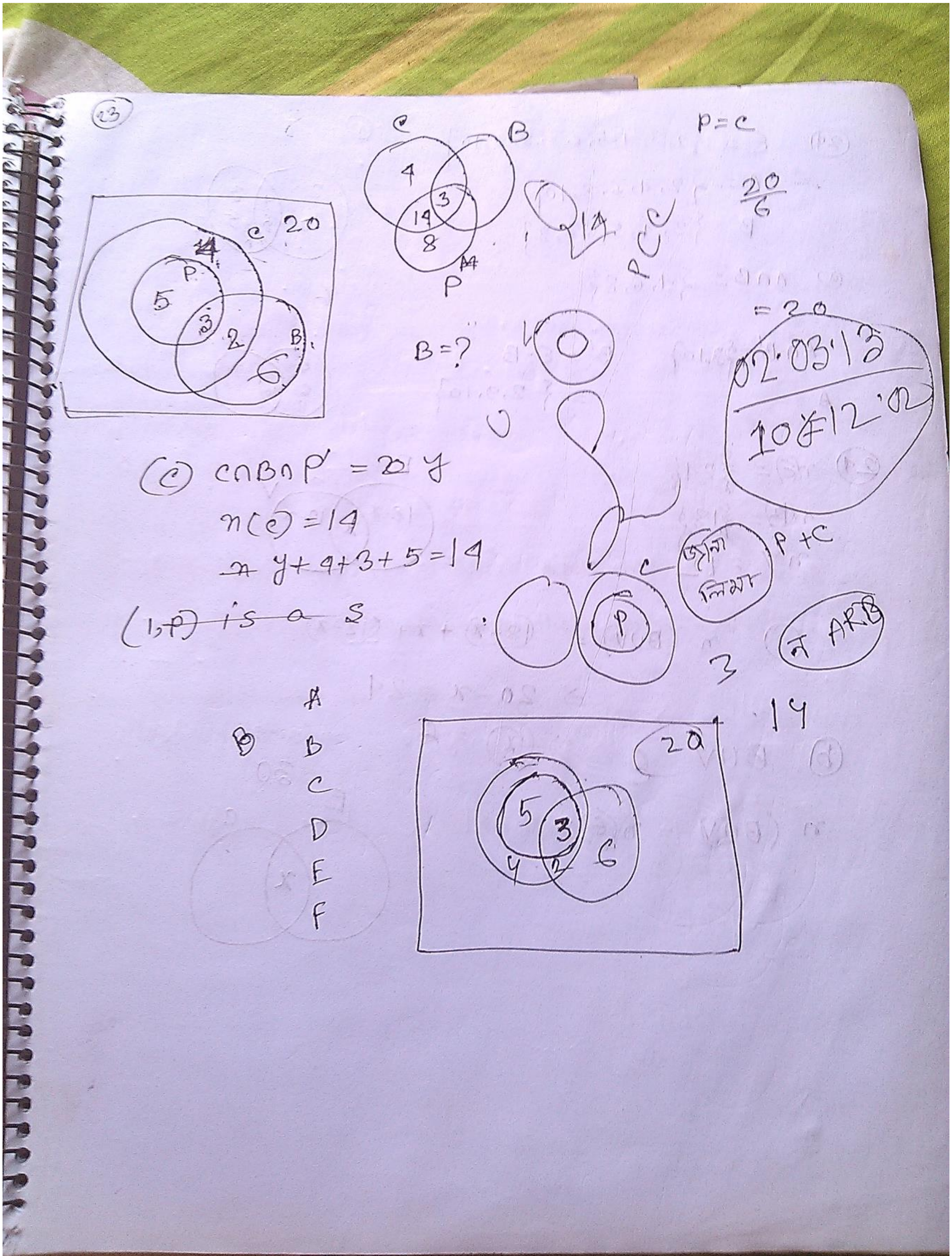
There are 32 students in a class and each studies at least one of the subjects: Geography or History. Of these 22 students study Geography and 15 study History. By drawing a venn diagram, find the number of student who study both History and Geography.



$22 - x + x + 15 - x = 32$

$37 - 32 = x$

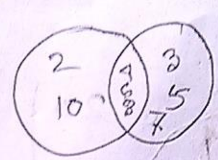
$x = 5$ Ans


23
C
B
P
P=C
20
14
B=?
02.03.13
10.12.12
C∩B∩P' = 20 y
n(C) = 14
y+4+3+5=14
(1,P) is a S
P+C
ARB
A
B
C
D
E
F
20
14

(20) $\mathcal{E} = \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$

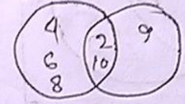
$A = \{2, 4, 6, 8, 10\}$

$B = \{3, 4, 5, 6, 7, 8\}$



(a) $A \cap B = \{4, 6, 8\}$

b



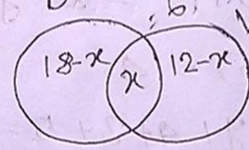
$A \cap B' = \{2, 10\}$ $B' = \mathcal{E} - B$

$= \{2, 9, 10\}$

(21) $n(\mathcal{E}) = \{24\}$

$n(B) = \{18\}$

$n V = 12$



(a) $n(B \cup V) = (18 - x) + x + (12 - x)$

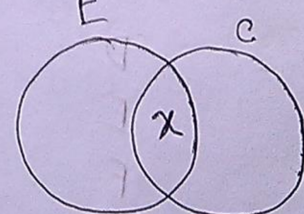
$\Rightarrow 30 - x = 24$

$\Rightarrow x = 6$

(b) $B \cup V = \mathcal{E}$

$n(B \cup V) = n(\mathcal{E})$

30

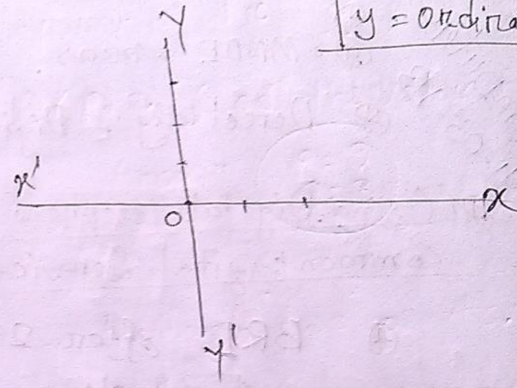


④ - Co-Ordinate Geometry :-

স্থানাঙ্কন

x = abscissa
y = Ordinate

Math ④ 11-03-13



Distance between two points :-

$P(x_1, y_1), \; Q(x_2, y_2)$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Ex 1. $P(-3, -2), \quad Q(-3, 5)$

$$PQ = \sqrt{(-3+3)^2 + (-2-5)^2}$$

$= 7$ unite.

Mid point এর কোন ফর্মুলা.

$P(x_1, y_1)$ ———— $P(x, y)$ ———— $Q(x_2, y_2)$

$$\text{Midpoind} = \left(\frac{x_1 + x_2}{2}, \; \frac{y_1 + y_2}{2} \right)$$

2.Q. M is the midpoint of the line joining the points A and B, Find:

Math. 11-07-13

(a) The co-ordinates of M if A and B have co-ordinates (−1, 2) and (3, −4) ~~Resp~~ respectively.

(b) The co-ordinates of A if M and B have co-ordinates (2, −1) and (3, 2) respectively.

Let
Answer to the Q. (a)

a) $M = \left(\frac{-1+3}{2}, \frac{2-4}{2}\right)$

$= \left(\frac{2}{2}, \frac{-2}{2}\right)$

$= (1, -1)$

A (−1, 2) (1, −1) B (3, −4)

$\frac{-1+3}{2}, \frac{2-4}{2}$ (1, −1)

~~b) $M \left(\frac{2+3}{2}, \frac{-1+2}{2}\right)$~~

~~$= \left(\frac{5}{2}, \frac{1}{2}\right)$~~

A (x, y) — M (2, −1) — B (3, 2)

Answer to the Q. (b)

b) $\frac{x+3}{2} = 2$

$x + 3 = 4$

$x = +1$

$\frac{y+2}{2} = -1$

$y + 2 = -2$

$y = -4$

A(x, y) . M(2, −1) B(3, 2)

$\frac{x+3}{2} = 2$

$x + 3 = 4$

$x = 1$

$\frac{y+2}{2} = -1$

$y + 2 = -2$

$y = -4$

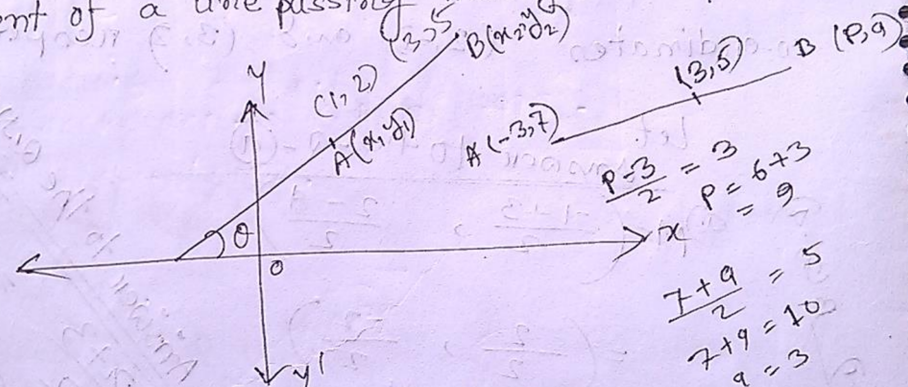
Assainment – 01

(Q-3) If M(3, 5) is the midpoint of the line joining the points A (-3, 7) and B(p, q) find the value of p and of q

Answer $p = 9$, $q = 3$

Math 11-03-17

Gradient of a line passing through two paints:-



$\frac{p-3}{2} = 3$

$p = 6+3$

$= 9$

$\frac{7+q}{2} = 5$

$7+q = 10$

$q = 3$

Gradient.

$m = \frac{\text{change in } y\text{-cordinate}}{\text{chage in } x\text{-cordinate}}$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{5-2}{3-1}$

$\frac{3}{2}$

① If A (1, 2) and B (3, 5), than. Gradient value

Answer. $\frac{5-2}{3-1} = \frac{3}{2}$

Q.No-2

② A (t, t+2), B (3t, 5t+2) - find the value of gradient?

Math 11-03-13

Answer:-

$\frac{5t+2-t-2}{3t-t}$

$= \frac{4t \cdot 2}{2t}$

= 2 Answer

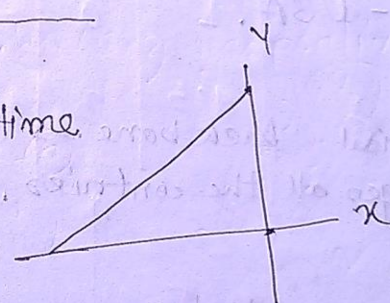
$\left(\frac{5t+2-t-2}{2}, \frac{3t-t}{2}\right)$

$\frac{4t}{2}, \frac{2t}{2}$

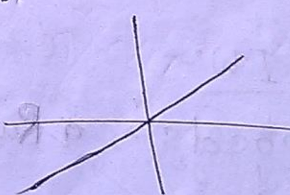
State line equation all time

$ax+by+c=0$

power 1



$y = mx + c$



$y = mx$

H the I. works:

Addressing Schemes:

13-03-13

navigete → ~~movie~~ Drama

Math

Math 13-03-13

At a point (x_1, y_1), the equation of straight line

$y - y_1 = m(x - x_1)$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

If $= \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$

1. find the equation of the line parsing through $(-3, 2)$ wit gradient 2.

$(y - 2) = 2(x + 3)$

$\Rightarrow y - 2 = 2x + 6$

$\Rightarrow 2x - y + 8 = 0$ Ans

$y - 2 = 2(x+3)$

$y - 2 = 2x + 6$

$\Rightarrow y - 2x = 8$

$(y - 3) = \frac{1}{2}(x + 1)$

$2y - 6 = -x + 1$

$2y + x = 5$

2. Find the eqn of the line parsing through the pair of paints $A(-1, 3)$, $B(1, 2)$

$$\frac{y - 3}{2 - 3} = \frac{x + 1}{1 + 1}$$

$\Rightarrow x + 2y - 5 = 0$

$\Rightarrow \frac{y - 3}{-1} = \frac{x + 1}{2}$

$\Rightarrow 2y - 6 = -x - 1 \Rightarrow$ ~~$x + y = 8$~~ ~~$x + 2y = -5$~~

* Find where the lines cut the x-axis and the y-axis. Hence sketch these lines.

Math. 13-03-13

$2y = 3x + 6$

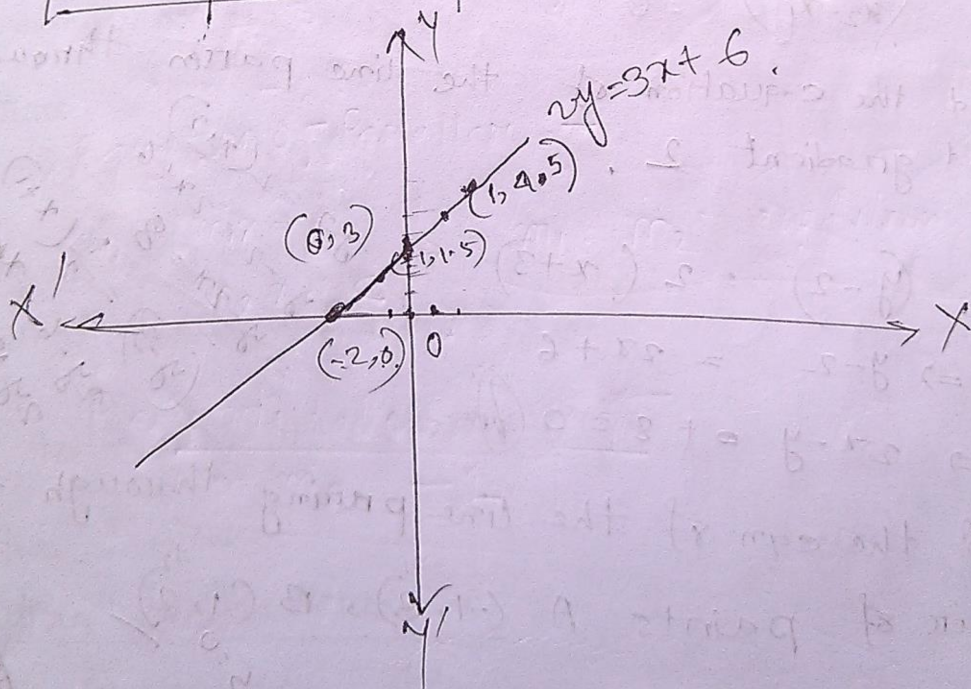
$y = \frac{3}{2}x + 3$

$-3x + 2y = 6$

$-\frac{x}{2} + \frac{y}{3} = 1$

$(-2, 0), (0, 3)$

x	1	0	−1	2	−2
y	4.5	3	1.5	6	0



The lines cut the x-axis at (−2, 0) the y-axis (0, 3)

① Assignment :-

Find the lines cut the x-axis and the y axis. Hence sketch these lines.

$4x - 3y - 12 = 0$

Math. 13-03-13

$\frac{x}{a} + \frac{y}{b} = 1$

$4x - 3y = 12$

$\Rightarrow \frac{4x}{12} - \frac{3y}{12} = 1$

$\Rightarrow \frac{x}{3} + \frac{y}{-4} = 1$

$(3, 0), (0, -4)$

The line joining the points A $(-1, 3)$ and B $(5, 15)$ meets the axes at P and Q. Find the equation of AB and calculate of PQ.

$\frac{y-3}{15-3} = \frac{x+1}{5+1}$

$\Rightarrow \frac{y-3}{12} = \frac{x+1}{6}$

$\Rightarrow 2x + 2 - y + 3 = 0$

$\Rightarrow 2x - y = -5$

$P\left(-\frac{5}{2}, 0\right), Q(0, 5)$

$\sqrt{\left(-\frac{5}{2}\right)^2 + (5)^2} = \frac{5\sqrt{5}}{2}$

Math.
13-03-13

The line –

$$PQ = \sqrt{\left(-\frac{5}{2}+0\right)^2 + (0+5)^2}$$

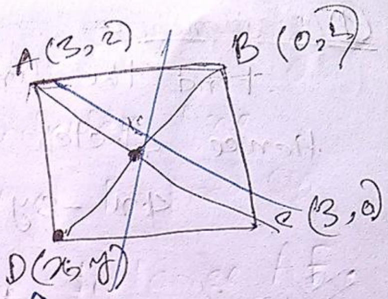
$$= \sqrt{\frac{25}{4} + 25}$$

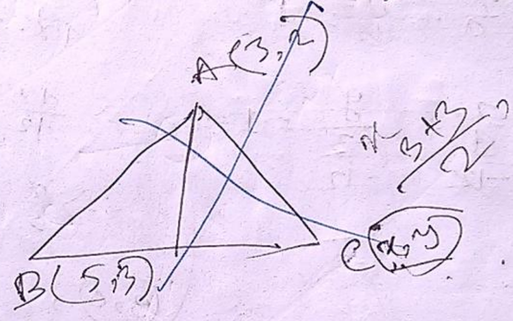
$$= \sqrt{\frac{25+100}{4}}$$

$$= \sqrt{\frac{125}{4}}$$

$$= \frac{\sqrt{5 \times 25}}{2}$$

$$= \frac{5\sqrt{5}}{2} \text{ A.}$$





Equation of Parallel:–

$$\left.\begin{aligned} y &= m_1 x + c \\ y &= m_2 x + c \end{aligned}\right\} \quad \boxed{m_1 = m_2 \text{ condition}}$$

Perpendicular. লম্ব

$$\boxed{m_1 m_2 = -1}$$

Math 13-03-13

⊛ Find the equation of the line which passes through the point A (2, 1) and it's parallel to the line $2x+3y=5$

$$y = -\frac{2}{3}x + \frac{5}{3} \quad \text{---(i)}$$

$$y = mx + c \quad \text{---(ii)}$$

(i) and (ii) →

$$m = -\frac{2}{3}$$

$$c = \frac{5}{3}.$$

$$(y - y_1) = m(x - x_1)$$

$$\Rightarrow (y - 1) = -\frac{2}{3}(x - 2)$$

$$\Rightarrow y - 1 = -\frac{2x}{3} + \frac{4}{3}$$

$$\Rightarrow 2x + 3y - 7 = 0$$

(157 page)

Assignment: ②

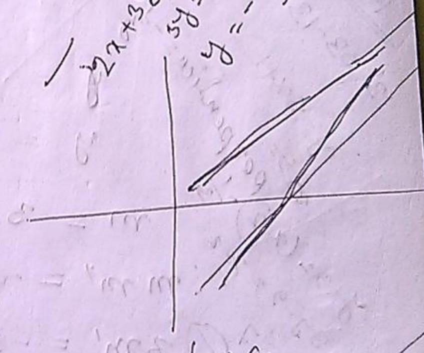
A (3, −2)

$$2x - 3y - 2 = 0$$

$$2x + 3y = 5$$

$$3y = -2x + 5$$

$$y = -\frac{2}{3}x + \frac{5}{3}$$



$$y + 2 = \frac{2}{3}(x - 3)$$

$$\Rightarrow 3y + 6 = 2x - 6$$

$$\Rightarrow 3y - 2x = -12$$

$$\Rightarrow 3y - 2x + 12 = 0$$

$$2x - 3y - 2 = 0$$

$$-3y = -2x + 2$$

$$y = \frac{2}{3}x - \frac{2}{3}$$

Q. Find the equ of the straight line passing through A (2, -3) and perpendicular line y = 3x+1. The two lines intersect at F. Find (i) the coordinates of F (ii) the distance AF.

Math
13-03-13

$y = 3x + 1$ —— (i)

$y = mx + c$

$m = 3$

$m m' = -1$

$3m' = -1$

$m' = -\frac{1}{3}$

$y + 3 = -\frac{1}{3}(x - 2)$

$3y + 9 = -x + 2$

$\Rightarrow x + 3y = -7$ —— (ii)

$\frac{x}{-7} + \frac{y}{-\frac{7}{3}} = 1$

$x = (-7 - 3y)$

$y = (-7 - 3y)3 + 1$

$= -21 - 9y + 1$

$10y = -20$

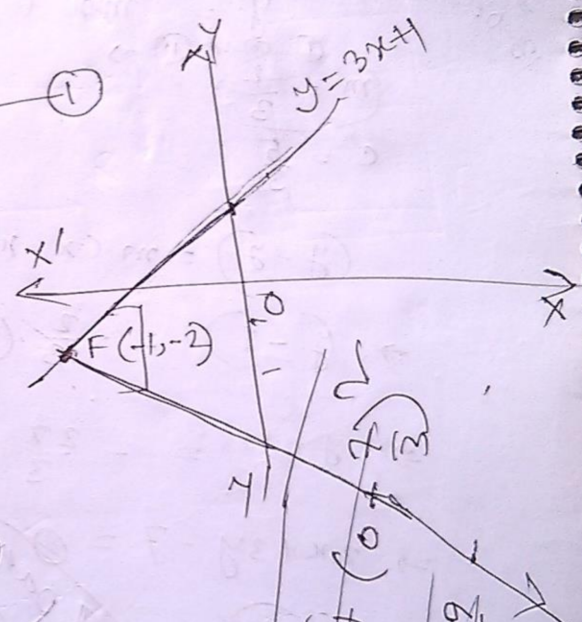
$y = -2$

$-2 - 1 = 3x$

$-3 = 3x$

$x = -1$

A(-1, -2)



$AF = \sqrt{(-7-0)^2 + \left(0 + \frac{7}{3}\right)^2}$

$= \sqrt{49 + \frac{49}{9}}$

Math 13-03-13

(ii) × 3 − (iii) →

$3x - y = -1$

$-3x \pm 9y = \mp 21$

$x = -1$

$\therefore y = -2$

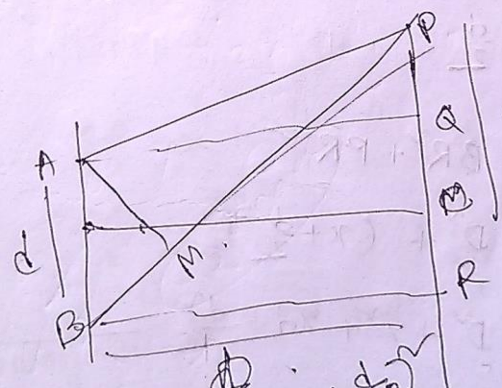
$F = (-1, -2)$

$AF = \sqrt{(2+1)^2 + (-3+2)^2}$

$= \sqrt{9 + 1}$

$= \sqrt{10}$

$4) 40 ($



$BP^2 = D^2 + (x + \frac{d}{2})^2$

$BP^2 - AP^2 = 2xd$

$(BP + AP)(BP - AP) = 2xd$

$BP - AP = \frac{2xd}{BP + AP}$

$BP - AP = \frac{2xd}{2D}$

$= \frac{xd}{D}$

P. d = $\frac{2D}{x} \times \frac{xd}{2D}$

Bright fringe

$\frac{xd}{D} = n\lambda$

$\therefore x = \frac{n\lambda D}{d}$

$x = \frac{(n+1)\lambda D}{d}$

* Given that $x^3 - 2x^2 + 5 = ax(x-1)^2 + b(x-1) + c$ for all values of x find the values of a, b and c.

Solution:

Since, $x^3 - 2x^2 + 5 = ax(x-1)^2 + b(x-1) + c$ for all values of x, the equation holds for any value of x.

Let $x = 1$ Then $1 - 2 + 5 = c$

$c = 4$

let $x = 0$ Then $5 = -b + c$

$b = -1$

let $x = 2$ Then $8 - 8 + 5 = 2a + b + c$

$5 = 2a + (-1) + 4$

$a = 1$

~~Altir~~ Alternatively, $x^3 - 2x^2 + 5 \equiv ax(x^2 - 2x + 1) + b(x-1) + c$

$\equiv ax^3 - 2ax^2 + ax + bx - b + c$

$\equiv ax^3 - 2ax^2 + (a+b)x + c - b$

Since the two polynomials are identical, the coefficients of every like power of x must be equal.

Equating the coeffients of

x^3: $a = 1$

x^1: $a + b = 0$

$1 + b = 0$

$b = -1$

x^0 (i,e. constant term):

$c - b = 5$

$\Rightarrow c + 1 = 5$

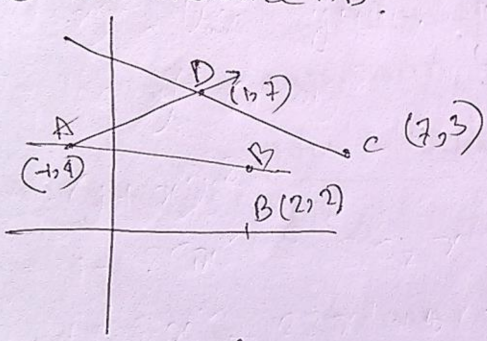
$\Rightarrow c = 4$

20-03-23

Two points have co-ordinates A (-1, 4), B (2, 2) and C(7, 3)

Find (a) the eqn of the line through A perpendicular to AB (b) the eqn. of the line through C parallel to AB (c) the co-ordinates of the point D at which there two lines intersect.

(d) the distance AD.



(a) $y - 4 = \frac{-1}{\left(\frac{-1-2}{4-2}\right)}(x-4)$

$\Rightarrow y - 4 = -\frac{+3}{2}(x-4)$

$\Rightarrow -2y + 8 = 3x - 3$

$\Rightarrow -2y = 3x + 11$

(b) $(y-2) = -\frac{2}{3}(x-2)$

$3y - 6 = -2x + 4$

$3y + 2x = 10$

(a) Gradiant of AB, $m = \frac{-2}{3}$

Gradiant of the perpendicular line, $m' = \frac{3}{2}$

(7, 3)

$\therefore$ Eqn of the perpendicular line through A is $y - 4 = \frac{3}{2}(x+1)$

$\Rightarrow 2y = 3x + 11$ —— (i)

(a) $y - 4 = \frac{3}{2}(x+1)$

$2y - 8 = 3x + 3$

$\Rightarrow 2y - 3x = 11$ —— (i)

(b) $y - 3 = \left(-\frac{2}{3}\right)(x - 7)$

$\Rightarrow 3y - 9 = -2x + 14$

$\Rightarrow 3y = -2x + 23$ —— (ii)

(b) $(y-3) = -\frac{2}{3}(x-7)$

$\Rightarrow 3y - 9 = -2x + 14$

$\Rightarrow 3y + 2x = 23$ —— (ii)

(c) $3\left(\frac{3x+11}{2}\right) = -2x+23$

$\Rightarrow \frac{9x+33}{2} = -2x+23$

$\Rightarrow 9x+33 = -4x+46$

$\therefore x = 1$

$\because 2y = 3+11$

$y = \frac{14}{2}$

$= 7$

$\therefore$ (d) $D(1, 7)$

(D) $AD = \sqrt{(1+1)^2 + (4-7)^2}$

$= \sqrt{4+9}$

$= \sqrt{13}$

Binomeal Theorem

$(1+b)^n = 1 + {}^nC_1 b^1 + {}^nC_2 b^2 + {}^nC_3 b^3 + \cdots b^n$

Q. Find the first 4 terms in the binomeal expression of (a) $(4x+1)^6$

(b) $(1-3x)^7$

(a) $(1+4x)^6$

$= 1 + 6C_1(4x) + 6C_2(4x)^2 + 6C_3(4x)^3 + \cdots$

$= 1 + 6\times 4x + 15\times 16x^2 + 20\times 64x^3 + \cdots$

$= 1 + 24x + 240x^2 + 1280x^3 + \cdots$

$13y = 91$

$y = 7$

$2\times 7 - 3x = 11$

$14 - 11 = 3x$

$(1, 7)$

(b) $(1-3x)^7$

$= 1 + 7C_1(-3x) + 7C_2(-3x)^2 + 7C_3(-3x)^3 + \cdots$

$= 1 - 7\times 3x + 21\times(9x^2) - 35\times(27x)^3$

$= 1 - 21x + 189x^2 - 945x^3 + \cdots$

* Find the first 4 terms in the expression of $(1+x^2)^8$. Use your result to estimate the value of $(1.01)^8$

$(1+x^2)^8$

$= 1 + 8C_1x^2 + 8C_2x^4 + 8C_3x^6 + \cdots$

$= 1 + 8x^2 + 28x^4 + 56x^6 + \cdots$

Comparing $(1.01)^8$ with $(1+x^2)^8$

$1 + x^2 = 1.01$

$x = \pm 0.1$

$(1+x^2)^8$

$(1.01)^8 = [1 + (0.1)^2]^8$

$= 1 + 8C_1(0.1)^2 + 8C_2(0.1)^4 + 8C_3(0.1)^6 + \cdots$

$= 1 + 0.08 + 0.0028 + 0.000056 + \cdots$

$= 1.082856$

A-4

Find ... $(1+2x)^9$ ... $(1.02)^9$

\# Find the first 3 terms in the expression of $(1-2x)^5$ and $(1+3x)^9$. Hence find the expansion of $(1-2x)^5(1+3x)^9$ up to terms in x^2.

$(1-2x)^5$

$= 1 + {}^5C_1(-2x) + {}^5C_2(-2x)^2$

$= 1 - 10x + 40x^2 + \ldots$

$(1+3x)^9$

$= 1 + {}^9C_1\, 3x + {}^9C_2\,(3x)^2$

$= 1 + 27x + 324x^2 + \ldots$

$(1-2x)^5(1+3x)^9 = (1-10x+40x^2-\ldots)(1+27x+324x^2+\ldots)$

$= 1+27x+324x^2-10x(1+27x)+40x^2+\ldots$

$= 1+27x+364x^2-10x-270x^2+\ldots$

$= 1+94x^2+17x+\ldots$

$(1-2x)^5 = 1 + {}^5C_1(-2x) + {}^5C_2(-2x)^2 + \ldots$

$= 1 - 10x + 20x^2 + \ldots$

$(1+3x)^9 = 1 + {}^9C_1\, 3x + {}^9C_2\,(3x)^2 + \ldots$

$= 1 + 15x + 324x^2 + \ldots$

$(1-2x)^5(1+3x)^9$

$(1-10x+20x^2-\ldots)(1+15x+324x^2+\ldots)$

* In the expansion of $(1+b)^n$, $T_{r+1} = {}^nC_r\, b^r$

Q. Find the terms in x^2 and x^5 in the expansion of $(1-x/2)^{12}$. Hence find the coefficient of x^5 in the expansion of $(3+2x^3)(1-\frac{x}{2})^{12}$

Sol: $(1-\frac{x}{2})^{12} = \left[1+\left(-\frac{x}{2}\right)\right]^{12}$

the term in x^r is $T_{r+1} = {}^{12}C_r\left(-\frac{x}{2}\right)^r$

for x^2, $T_3 = {}^{12}C_2\left(-\frac{x}{2}\right)^2$

$= 66 \times \frac{1}{4}(-x)^2$

$= 66\frac{x^2}{4}$

$= \frac{33x^2}{2}$

on 1TV Live. in

for x^5, $T_6 = {}^{12}C_5\left(-\frac{x}{2}\right)^5$

$= 198\,(-x^5) \times \left(-\frac{1}{2}\right)^5$

$= -\frac{99}{4}x^5$

$(3+2x^3)\ldots\left[\ldots + \frac{33}{2}x^2 - \frac{99}{4}x^5 + \ldots\right]$

$= \frac{-3\times 99}{4}x^5 + 2x^3\,\frac{33}{2}x^2 \ldots$

$= \frac{-297}{4}x^5 + \frac{66}{2}x^5 + \ldots$

$= \frac{-165}{4}x^5 + \ldots$

$\therefore$ coefficient of x^5 is .

$-\frac{165}{4}x^5$ Ans.

(A-5) Find the expansion of $(1-x)^6$ and $(1+2x)^6$ as far as the terms in x^3. Hence expand $(1+x-2x^2)^6$ up to terms in x^3.

Formula:

$(a+b)^n = a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + {}^nC_3 a^{n-3} b^3 + \cdots b^n$

$\because$ Given that $(p-\frac{1}{2}x)^6 = r - 96x + 5x^2 + \cdots$ Find p, r and

$\Rightarrow (p-\frac{1}{2}x)^2 = p^6 + 6C_1 p^5(-\frac{1}{2}x) + 6C_2 p^4(-\frac{1}{2}x)$

$= p^6 - 3p^5 x + \frac{15}{4} p^4 x^2 + \cdots$

$p^6 - 3p^5 x + \frac{15}{4} p^4 x^2 = r - 96x + 5x^2$

$\Rightarrow$ Comparing the co-efficients

$p^6 = r$

$-3p^5 = -96$

$\Rightarrow p^5 = 32$

$p^5 = 2^5$

$p = 2$

$r = 64$

$s = \frac{15}{4} p^4$

$= \frac{15}{4} \times 2^4$

$= 60$

$p = 2, r = 64, s = 60$

Q. Write down first three terms in the expansion of $(2+ax)^5$. Given that the first three terms in the expansion of $(b+2x)(2+ax)^5$ are $96 - 176x + cx^2$. Find the values of a, b, and c

A. $(2+ax)^5 = 2^5 + 5c_1 2^4 (ax) + 5c_2 \cdot 2^3 \cdot a^2 \cdot x^2 + \dots$

$= 32 + 80ax + 80a^2x^2$

Given that,

$(b+2x)(2+ax)^5 = 96 - 176x + cx^2$

$\Rightarrow (b+2x)(32 + 80ax + 80a^2x^2) = 96 - 176x + cx^2$

$\Rightarrow 32b + 80abx + 80a^2bx^2 + 64x + 160ax^2 + 160a^2x^3 = 96 - 176x + cx^2$

$\Rightarrow 32b + (80ab + 64)x + 80(a^2b + 160a)x^2 = 96 - 176x + cx^2$

Comparing the coefficients.

$32b = 96$

$b = 3$

Now, $80ab + 64 = -176$

$a = -1$

and $80a^2b + 160a = c$

$\Rightarrow 80\times3 - 160 = c$

$c = 80$

$a = -1,\ b = 3,\ c = 80$ Ans

(A-5)

$(1-x)^6 = 1 + 6C(-x) + \ldots$

$(2+ax)^5 = 2^5 + {}^5C_1 \cdot 2^4 \cdot ax + {}^5C_2\, 2^3 \cdot a^2x^2 + \ldots$

$= 32 + 80ax + 80a^2x^2$

$(b+2x)(2+ax)^5 =$

$= (b+2x)(32 + 80ax + 80a^2x^2 + \ldots)$

$= 32b + 80abx + 80a^2bx^2 + 64x + 160ax^2 + 160a^2x^3 + \ldots$

$= 32b + x(80ab+64) + 80(a^2b + 2a)x^2 + \ldots$

$96 = 32b$

$b = 3$

$80ab + 64 = -176$

$80\times3a = -176 - 64$

$a = -1$

$80a^2b + 160a = c$

$80\cdot1\cdot3 + 160\times(-1) = c$

$c = 80$

Binomial Theorem

In the expanstion of $(a+b)^n$, $T_{r+1} = {}^nC_r\, a^{n-r}\, b^r$

In each of the following expansions, find the indicated term

(a) $\left(2+\frac{x^2}{2}\right)^{11}$, 7 term

(b) $\left(2x-\frac{1}{x^2}\right)^{12}$; term in $\frac{1}{x^{15}}$

$T_{r+1} = {}^{11}C_r\, a^{r-6}\, b^r$

$= {}^{11}C_r\, 2^{r-6}\, b^r$

$= {}^{11}C_r\, 2^{r-6} \cdot \left(-\frac{1}{x^2}\right)^r$

$= {}^{11}C_r \cdot 2^{r-6} \cdot (-1)^r \cdot (x^{-2})^r$

$T_{6+1} = T_7 = {}^{11}C_6\, 2^5\, (x^{-2})^6$

$= 231\, x^{12}$

$T_{r+1} = {}^{12}C_r \cdot (2x)^{12-r} \cdot \left(-\frac{1}{x^2}\right)^r$

$= {}^{12}C_r \cdot 2^{12-r}\, (-1)^r \cdot x^{12-3r}$

$12-3r = -15$

$3r = 12+15$

$3r = 27$

$r = 9$

$\frac{1}{x^{15}}$ ${}^{12}C_9$

${}^{12}C_9\, 2^3\, (-1)^9\, x^{12-27}$

$= -\frac{1760}{x^{15}}$ Ans

$\left(2+\frac{x^2}{2}\right)^{11}$

$T_{r+1} = {}^{11}C_r \cdot 2^{11-r} \cdot x^{2r} \cdot 2^{-r}$

$T_{6+1} = {}^{11}C_6\, 2^5 \cdot x^{12} \cdot 2^{-6}$

$= 231\, x^{12}$ Ans

(b) $\left(2x - \frac{1}{x^2}\right)^{12}$ x^{15}

$T_{r+1} = {}^{12}C_r (2x)^{12-r} \cdot \left(-\frac{1}{x^2}\right)^r$

power of in

$T_{r+1} = x^{12-r} \quad 12 - r - 2r$

$= 12 - 3r$

$\frac{12+15}{3} = \frac{27}{3} = 9$

$\therefore T_{r+1}$ for x^{-15}

$\therefore 12 - 3r = -15$

$-3r = -15 - 12$

$\therefore r = \frac{27}{3}$

$= 9$

the term in $\frac{1}{x^{15}} = {}^{12}C_9 (2x)^3 \cdot (-x^{-2})^9$

$\therefore$ the term in $\frac{1}{x^{15}} = {}^{12}C_9 (2x)^3 (-x^{-2})^9$

$= -\frac{1760}{x^{15}}$ Ans

$\therefore x = \frac{55}{8192}$

(A.7) In the expansion of $\left(x^3 - \frac{2}{x^2}\right)^{10}$, (3823 Exm-12, A-6)

find (a) the term in x^{10}

(b) the coefficient of $\frac{1}{x^5}$

$\frac{30}{5}=$ (c) the 6th term.

x^{-1-1}

x^{-2}

(335 page)

Differentitation

Given that the curve $y = ax^2 + \frac{b}{x}$ has gradient 4 at point (1, 5), calculate the value of a and b

$y = ax^2 + \frac{b}{x}$ —— (i)

Since the point on the curve

$\Rightarrow 5 = a \cdot 1^2 + \frac{b}{1}$

$\Rightarrow 5 = a + b$ —— (ii)

$\frac{dy}{dx} = 2ax + b(-x)^{-2}$

i.e $\frac{dy}{dx} = 4$

$\Rightarrow 4 = 2ax - bx^{-2}$

~~$\Rightarrow 4 = 2a - b$~~

$\Rightarrow 4 = 2a \cdot 1 - b$

$\Rightarrow 2a - b = 4$ —— (iii)

$3a = 9$

$a = 3$

$3 + b = 5$

$b = 2$

$\therefore \left.\begin{matrix} a = 3 \\ b = 2 \end{matrix}\right\}$ Ans

$y = (3x^2+2)^7$

$\frac{dy}{dx} = 7(3x^2+2)^6 \cdot 6x$

$= 42x(3x^2+2)^6$

$y = 2(\sqrt{x}+2)^{1/2}$

$\frac{dy}{dx} = 2 \cdot \frac{1}{2}(\sqrt{x}+2)^{\frac{1}{2}-1} \cdot \frac{1}{2}x^{(\frac{1}{2}-1)}$

$= (\sqrt{x}+2)^{-\frac{1}{2}} \cdot \frac{1}{2}x^{\frac{1-2}{2}}$

$= (\sqrt{x}+2)^{-\frac{1}{2}} \cdot \frac{1}{2} \cdot x^{-\frac{1}{2}}$

$= \frac{1}{2\sqrt{x}(\sqrt{x}+2)^{1/2}}$

$T_{r+1} = {}^{10}C_r \cdot (x^3)^{10-r} \cdot \left(-\frac{2}{x^2}\right)^r$

$= {}^{10}C_r \cdot (-2)^r \cdot x^{30-5r}$

(a) $30-5r = 10$

$5r = 20$

$r = 4$

the term in $x^{10} = {}^{10}C_4 \cdot (-2)^4 \cdot x^{30-20}$

$= 3360x^{10}$

$5 = a + b$ — (1)

$\frac{dy}{dx} = 2ax - 6x^{-3}$

$4 = 2a - b$

$a = 3$

$b = 2$.

Calculate the co-ordinates of the point in the curve $y = \sqrt{x^2 - 2x + 5}$ at which $\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{1}{2}\left(x^2 + 2x + 5\right)^{-\frac{1}{2}} \cdot (2x - 2) = 0$$

$\Rightarrow$ ~~$x^2 + 2x + 5 = 0$~~

$$\Rightarrow x - 1 = 0$$

$$x = 1$$

$$\therefore y = \sqrt{1 - 2 + 5} = \sqrt{4} = 2$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x^2 - 2x + 5}} \cdot (2x - 2)$$

$$2x - 2 = 0$$

$$x = 1$$

$$y = \sqrt{1 - 2 + 5} = 2$$

$$y = f(x) = \sqrt{1 + \sqrt{x}}$$

$$\frac{dy}{dx} = \frac{1}{2}\left(1 + \sqrt{x}\right)^{\frac{1}{2} - 1} \cdot \left(0 + \frac{1}{2}x^{-\frac{1}{2}}\right)$$

$$= \frac{1}{2}\left(1 + \sqrt{x}\right)^{\frac{1}{2} - 1} \cdot \frac{1}{2\sqrt{x}}$$

$$f'(x) = \frac{1}{4\sqrt{x}\left(1 + \sqrt{x}\right)}$$

$$= \frac{1}{4\sqrt{x\left(1 + \sqrt{x}\right)}}$$

$$y = f(x) = \sqrt{1 + \sqrt{x}}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1 + \sqrt{x}}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{4\sqrt{x(1 + \sqrt{x})}}$$

Q) $y = x^2 - 3x + 4$

$$\frac{dy}{dx} = 2x - 3 = 2 - 3 = -1$$

$$y - 2 = -x - 1$$

$$y = -x + 3$$

$$y - 4 = \frac{1}{1}(x - 3)$$

$$y - 4 = x - 3$$

$$y - x = 1$$

$y = \frac{\sqrt{1-x}}{\sqrt{x^2+3}}$

$= \frac{-u\frac{dv}{dx} + v\frac{du}{dx}}{v^2}$

$= -\sqrt{1-x} \cdot \frac{d}{dx}$

Atangent

Database only memory

but,

01-04-2013

15.5 page No - 346,

Math

Equation of taugent and normal :

স্পর্শক The equation of tangent is $y - y_1 = m(x - x_1)$

অভিলম্ব The equation of normal is $y - y_1 = -\frac{1}{m}(x - x_1)$

Example-15

01-04-13

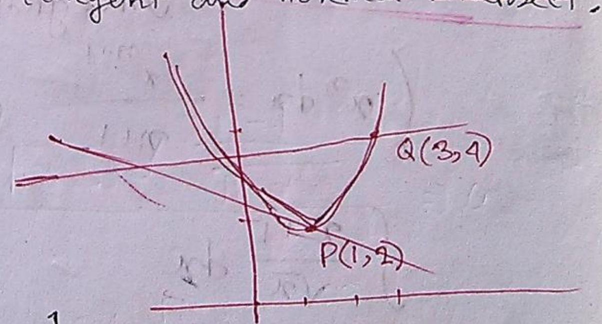
The curve বক্ররেখা $y = x^2 - 3x + 4$ passes through the points P (1, 2) and Q (3, 4).

Find (a) the equation of tangent at P,

(b) the equation of the normal at Q,

(c) the co-ordinates of R the point of ~~intersection~~ intersection

values where the tangent and normal ~~interseet~~ intersect.



(a) $y = x^2 - 3x + 4$

$\frac{dy}{dx} = 2x - 3$

at P (1, 2), $\frac{dy}{dx} = 2 - 3 = -1$

Equation of the tangent at p is $(y-2) = -1(x-1)$

$\Rightarrow y - 2 = -x + 1$

$\Rightarrow y + x - 3 = 0$

$\Rightarrow y = -x + 3$ ——— (1)

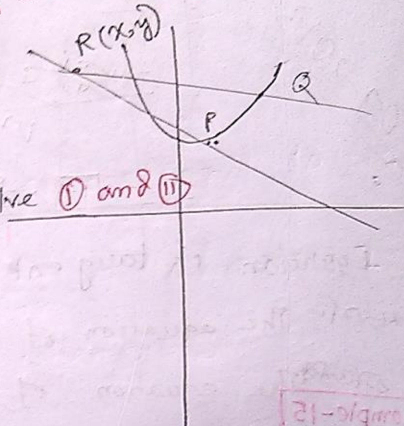
(347)

Math. 01-04-13

(b) at (3, 4), $\frac{dy}{dx} = 6 \cdot \frac{3}{3} = 3$

Gradient of the normal at Q is $-\frac{1}{3}$ and its equation is

$$(y - 4) = -\frac{1}{3}(x - 3)$$



$$3y - 12 = -x + 3$$

$$3y = -\frac{1}{3}x + 15 \quad \text{—— (ii)}$$

(c). For the point R, we solve (i) and (ii)

(i) − (ii) → (1) and (2)

$$3y - y = -15 + 3$$

$$2y = 12$$

$$y = 6$$

$$6 = -x + 3$$

$$x = 3 - 6 = -3$$

$$(x, y) = (-3, 6)$$

The coordinates of the point R are (−3, 6) (Ans)

Integrati

$$\int x^n dx = \frac{x^{n+1}}{n+1} c.$$

$$\int \frac{u}{v} = v\frac{d}{dx}u +$$

$$\int \frac{x^2+1}{\sqrt{x}} dx$$

$$= \int \frac{x^2}{\sqrt{x}} dx + \int \frac{1}{\sqrt{x}} dx$$

$$= \int x^{3/2} dx + \int x^{-1/2} dx$$

Math. 01-07-13

(1) Given, $\frac{dy}{dx}$ is inversely proportional to x^2 and that $y=1$ and $\frac{dy}{dx}=3$ when $x=3$ find the value of y when $x=2$.

(1) $\frac{dy}{dx} \propto \frac{1}{x^2}$

$\Rightarrow \frac{dy}{dx} = k/x^2$

$3 = k\frac{1}{9}$

$k = 27$.

$\frac{dy}{dx} = \frac{27}{x^2}$

$\int dy = \int \frac{27}{x^2} dx$

$= 27\int \frac{1}{x^2} dx$

$= 27 \cdot \frac{x^{-2+1}}{-1} + c$

$y = -27 \cdot \frac{1}{x} + c$

when.

$x=3$

$y=1$

$1 = -\frac{27}{3} + c$

$c = 10$

$\frac{dy}{dx} \propto \frac{1}{x^2}$

$\frac{dy}{dx} = \frac{k}{x^2}$

$3 = \frac{k}{9}$

$k = 27$

$\frac{dy}{dx} = \frac{27}{x^2}$

$dy = \frac{27}{x^2} dx$

$y = \int x^{-2} dx$

$y = -27\frac{1}{x} + c$

$1 = -9 + c$

$c = 10$

$y = -\frac{27}{x} + 10$

$x = -27/2$

$y = -\frac{27}{x} + 10$ 2) 27 (13

$= -\frac{27}{2} + 10$

$= -\frac{7}{2}$

(A-8) Given $\frac{dy}{dx}$ is directly proportional to (x^2-1)

334-(12) and that $y=3$. and $\frac{dy}{dx} = 9$ when $x=2$

find the value of y when $x=3$

$$\int \frac{1}{(2x-3)^2}\,dx$$

$$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a\cdot(n+1)}$$

$$\frac{(2x-3)^{-2+1}}{-1}$$

$$-(2x-3)^{-1} \qquad \int (2x-3)\,dx$$

$$= \frac{1}{(2x-3)} \qquad = \frac{x^2}{2} - 3x$$

$$= \frac{-x^2}{2x-3} \qquad (x^2-3x)$$

$$\int (2x-\sqrt{x})^2\,dx$$

$$\frac{(2x-\sqrt{x})^3}{3} \qquad \int (2x-\sqrt{x})\,dx$$

Math, 01-07-13

$\frac{dy}{dx} \propto (x^2-1)$

$\Rightarrow \frac{dy}{dx} = k(x^2-1)$

$9 = k3$

$k = 3$

$\frac{dy}{dx} = 3x^2 - 3$

$y = \frac{3x^3}{3} - 3x + c$

$3 = 2^3 - 3\times 2 + c$

$c = 1$

$y = x^3 - 3x + 1$

$= 3^3 - 3\times 3 + 1$

$= 19$

$$= \frac{(2x-\sqrt{3x})^3}{3} \cdot \left(2 \cdot \frac{x^2}{2} - \frac{x^{3/2}}{3/2}\right)$$

$$= \frac{(2x-\sqrt{3})^3}{3} \cdot \left(x^2 - \frac{2}{3}x^{3/2}\right)$$

$$\int \frac{4}{3(\sqrt{6x-1})} dx$$

$$= \frac{4}{3} \int (6x-1)^{-\frac{1}{2}} dx$$

$$= \frac{4}{3} \cdot$$

Math. 01-04-13

Integration.

Given that $y = (x-2)\sqrt{x+1}$ Show that $\frac{dy}{dx} = \frac{3x}{2\sqrt{x+1}}$

Hence evaluate $\int_3^8 \frac{x}{\sqrt{x+1}} dx$

$$\frac{dy}{dx} = \frac{1}{2}(x-2)(x+1)^{-1/2} + \sqrt{x+1}$$

$$= \frac{(x-2)}{2\sqrt{(x+1)}} + \sqrt{x+1}$$

$$\frac{dy}{dx} = \frac{3x}{2\sqrt{x+1}}$$

(437)

(437) Reversing the process of integration.

$$\int_3^8 \frac{2x}{2\sqrt{x+1}}\,dx = \left[(x-7)\sqrt{x+1}\right]_3^8$$

$$\Rightarrow 16 = \frac{3}{2}\int_3^8 \frac{x}{\sqrt{x+1}}$$

$$\frac{32}{3} = \int_3^8 \frac{x}{\sqrt{x+1}}$$

$\sqrt{x+1}$

$\int \frac{(x+1)^{3/2}}{\frac{3}{2}}$

$\frac{2}{3}(x+1)^{3/2}$

$(8-7)\sqrt{8+1}$ $-(3-7)\sqrt{(3+1)}$ $= 2\times 2 + 4\times 3$ $= 4 + 20$

Math - 01-04-13

$$\int \cos^2\left(\frac{\pi}{4}-2x\right)dx$$

$$= \frac{1}{2}\int 2\cos^2\left(\frac{\pi}{4}-2x\right)dx$$

$2\cos^2 x = 1+\cos 2x$

$$= \frac{1}{2}\int 1 + \int \cos\left(\frac{\pi}{2}-4x\right)dx$$

$\cos(90-4x)$

$\sin 4x\, dx$

$$= \frac{1}{2}\left[x + \int \sin 4x\, dx\right]$$

$-\frac{\cos 4x}{4}\Big] + C$

$\int e^{(1-3x)}$

03-04-13

$\frac{e^{1-3x}}{-3}$

$\int_0^{\pi/4} (\cos 2x - \sin x)\, dx.$

$\left[\frac{\sin 2x}{2} + \cos x\right]_0^{\pi/4}$

$\frac{\sin\frac{\pi}{2}}{2} + \cos\frac{\pi}{4} - \frac{\sin 0}{2} - \cos 0$

$2\cos^2 x = 1 + \cos 2x$

$= \frac{1}{\sqrt{2}} - \frac{1}{2}$

$\int \cos^2\left(\frac{\pi}{4} - 2x\right) dx$

$= \frac{1}{2}\int 2\cos^2\left(\frac{\pi}{4} - 2x\right) dx$

$= \frac{1}{2}\int \left\{1 + \cos\left(\frac{\pi}{2} - 4x\right)\right\} dx$

$= \frac{1}{2}\left\{x + \int \sin 4x\right\}$

$= \frac{1}{2}\left\{x - \frac{\cos 4x}{4}\right\} + c$

03-04-13

$$\int_1^3 \frac{1-4x+x^3}{2x^3}\,dx$$

$$= \int_1^3 \frac{1}{2x^3}\,dx - \int_1^3 \frac{4x}{2x^3}\,dx + \int_1^3 \frac{x^3}{2x^3}\,dx$$

$$= \frac{1}{2}\int_1^3 x^{-3}\,dx - 2\int_1^3 x^{-2}\,dx + \frac{1}{2}\int_1^3 1\,dx$$

$$= \left[\frac{1}{2}\times\frac{x^{-2}}{-2}\right]_1^3 - 2\left[-\frac{1}{x}\right]_1^3 + \frac{1}{2}\left[x\right]_1^3$$

$$= -3^{-2} - 2\left[\frac{1}{3} - 1\right] + \frac{1}{2}\left[3 - 1\right]$$

$$= -\frac{1}{9} - 2\left[\frac{1-3}{3}\right] + 1$$

$$= -\frac{1}{9} + \frac{4}{3} + 1 \qquad = -\frac{1}{9}\ \text{Ans}$$

Assi-9.

440 Page

$$\int_1^9 \frac{3-2\sqrt{x}}{x^2}dx$$

440

(11) A-10. Show that $\frac{d}{dx}\left(\frac{x}{1+2x}\right) = \frac{1}{(1+2x)^2}$. Hence evaluate $\int_1^4 \left(\frac{2}{1+2x}\right)^2 dx$.

(349 page-10)

(A-11) Find the equation of the tangent to the curve $y = 3x^2-2x+5$, which is perpendicular to the line $4y+x=2$

(Ans: $y = 4x+2$)

Ans;

$y = 3x^2-2x+5$ — (1)

$\frac{dy}{dx} = 6x-2$

$4y+x=2$

$y = -\frac{1}{4}x+2$

$-\frac{1}{4}\times(6x-2)=-1$

$x=1$

(1) No $x=1$ put $y=3-2+5$

$y=6$

$\left.\frac{dy}{dx}\right|_{x=1} = 6-2 = 4$

$(y-6) = 4(x-1)$

$y-6 = 4x-4$

$y-4x = 6-4$

$y = 4x+2$ (Ans)

$$\int_1^9 3x^{-2} - 2x^{\frac{1}{2}-2}dx$$

$$= \int_1^9 \left(3x^{-2} - 2x^{-3/2}\right)dx$$

$$= \left[-3x^{-1} + \frac{2 \cdot x^{-\frac{1}{2}}}{\frac{1}{2}}\right]_1^9$$

$$= \left[\frac{4}{\sqrt{x}} - \frac{3}{x}\right]_1^9$$

$$= \frac{4}{3} - \frac{3}{9} - 4 + 3$$

$$= 0$$

Inequaility

A.M → Arithmatic mean.

G.M → Geometric Mean.

A.M :- The arithmatic mean of any n peritive quantition in the nth part of the sound.

$$A.M = \frac{a_1 + a_2 + \cdots + a_n}{n}$$

G.M = The geometrix mean of any n paricle quantition, is the n th root of this product,

AR

$$G.M = (a_1, a_2 \cdots a_n)^{\frac{1}{n}}$$

$AM > GM$

If $2S = a+b+c$ prove the $S^3 > 27(S-a)(S-b)(S-c)$

Let, $x = S-a$

$y = S-b$

$z = S-c$

Since, $AM \geq GM$

$$= \frac{x+y+z}{3} > (xyz)^{1/3}$$

$$= \frac{x-a+s-b+s-c}{3} > \{(s-a)(s-b)(s-c)\}^{1/3}$$

$$\Rightarrow 3s-(a+b+c) > 3\{(s-a)(x-b)(x-c)\}^{1/3}$$

$$\Rightarrow 3s-2s > 3\{(s-a)(x-b)(x-c)\}^{1/3}$$

$$\Rightarrow s > 3\{(x-a)(x-b)(x-c)\}^{1/3}$$

$$\Rightarrow s^3 > 27(s-a)(s-b)(s-c)$$

[Proved]

If, a, b, c are positive integers, prove that $(b+c)(c+a)(a+b) > 8abc$ and hence prove that $xyz > (y+z-x)(z+x-y)(x+y-z)$

Since,

A.M. > G.M.

$$\frac{b+c}{2} > \sqrt{bc}$$

$$\frac{c+a}{2} > \sqrt{ca}$$

$$\frac{a+b}{2} > \sqrt{ab}$$

$$(b+c)(c+a)(a+b) > 8\sqrt{a^2b^2c^2}$$

$$(b+c)(c+a)(a+b) > 8abc \quad \text{—(1)}$$

let,

$a = y+z-x$

$b = z+x-y$

$c = x+y-z$

(1), No value,

$$(z+x-y+x+y-z)(x+y-z+y+z-x)(y+z-x+z+x-y) > 8(y+z-x)(z+x-y)(x+y-z)$$

$$2x \times 2y \times 2z > 8(y+z-x)(z+x-y)(x+y-z)$$

$$xyz > (y+z-x)(z+x-y)(x+y-z)$$ [Proved]

Assignment

(12) If the sum of any two of the equations x, y and z be greater than the third, show that,

$$(x+y+z)^3 > 27(y+z-x)(z+x-y)(x+y-z)$$

1. Q. If M (3,5) is the midpoint of the line going the points A (-3,7) and B (P,9) find the value of p and q.

2. Find the lines cut the x-axis and the y axis. Here sketch there lines, $4x-3y-12=0$

3. Find the equation of the line passing through the point (3,-2) and parallel to the line $2x-3y-2=0$

4. Obtain the first 4 terms in the expansion of $(1+2x)^9$ in ascending power of x. Use this expansion to find an approximate value of $(1.02)^9$.

5. Find the expansions of $(1-x)^6$ and $(1+2x)^6$ as far as the term in x^3, Hence expand $(1+x-2x^2)^6$ up to the term in x^3.

6. Write down in ascending power of x, the first three terms in the expansion of $(x+ax)^5$. Given that the first three terms in the expansion of $(b+2x)(2+ax)^5$ are $96-176x+cx^2$ find the value of a, b, c

07. In the expansion of $\left(x^3 - \frac{2}{x^2}\right)^{10}$, find

(a) the term in x^{10}

(b) the coefficient of $\frac{1}{x^5}$

(c) the 6th term.

(8) Given $\frac{dy}{dx}$ is directly proportional to (x^2-1) and that $y=3$ and $\frac{dy}{dx} = 9$, when $x=2$, find the value of y when $x=3$

(9) $\int_1^9 \frac{3-2\sqrt{x}}{x^2}\,dx$

(10) Show that $\frac{d}{dx}\left(\frac{x}{1+2x}\right) = \frac{1}{(1+2x)^2}$ Hence evaluate

$$\int_1^4 \left(\frac{2}{1+2x}\right)^2 dx$$

(11) Find the equation of the tangent to the curve, $y = 3x^2 - 2x + 5$, which is perpendicular to the line $4y + x = 2$

(12) If the sum of any two of the equation x, y and z be greater than the third, show that

$$(x+y+z)^3 > 27(y+z-x)(z+x-y)(x+y-z)$$